High Performance Computing for Operation Research

Claude TADONKI

IEF - Paris Sud University
claude.tadonki@u-psud.fr

INRIA-Alchemy seminar, Thursday March 17

Research topics

- Fundamental Aspects of Algorithms and Complexity (Formalism, proof, quantification, classification, ...)
- Discrete Dynamical Systems (Chip Firing Game, Periodicity, Garden of Eden, Invariants, ...)
- Parallel Scheduling (SPMD, SIMD, Systolic, ...)
- Polyhedral model (recurrences equations, re-indexation, systematic synthesis, code derivation, ...)
- Efficient Parallel Programming (Standard Parallel Machines et (Heterogeneous/Clustered) multicores machines)
- Program Transformations (Source-to-source framework aiming at monitoring parallel implementations)
- Power Aware Computing (Energy Efficient Embedded Programs)
- Applied Mathematics and Operation Research (Mathematical modeling, Optimization, Linear Algebra, Matrix Computation)

Discrete Dynamical Systems

Given a graph $G = (X, A)$ and a transition function φ such that $\varphi(G) = (X, A')$ with $card(A) = card(A')$. We study the evolution $G_0 = G$, $G_1 = \varphi(G_0)$, $\cdots$, $G_{n+1} = \varphi(G_n)$.

Remark: on a finite graph, such an evolution is ultimately periodic. Any state of the graph before the period is called transient.

Interesting questions are

- the set of period lengths for a given graph
- bounds of period lengths
- garden of Eden (graph with no transient length)
- relation between the structure of the graph and periodicity
- applications (games, simulation, graph scheduling)

This work gave an inspiration to derive a parallel scheduling methodology for acyclic graphs with some recurrent structure.

Research topics

- Fundamental Aspects of Algorithms and Complexity (Formalism, proof, quantification, classification, ...)
- Discrete Dynamical Systems (Chip Firing Game, Periodicity, Garden of Eden, Invariants, ...)
- Parallel Scheduling (SPMD, SIMD, Systolic, ...)
- Polyhedral model (recurrences equations, re-indexation, systematic synthesis, code derivation, ...)
- Efficient Parallel Programming (Standard Parallel Machines et (Heterogeneous/Clustered) multicores machines)
- Program Transformations (Source-to-source framework aiming at monitoring parallel implementations)
- Power Aware Computing (Energy Efficient Embedded Programs)
- Applied Mathematics and Operation Research (Mathematical modeling, Optimization, Linear Algebra, Matrix Computation)

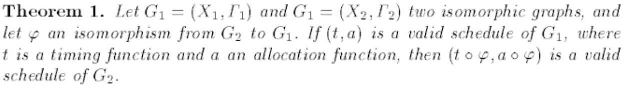
Polyhedral model and parallel scheduling

A **system of recurrence equations** (SRE) defining a variable X has the following form

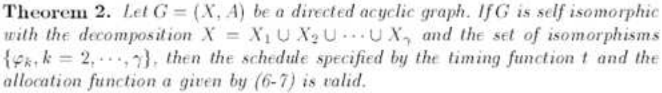
$$X(z) = \begin{cases} & \vdots \\ D_i^X & : \quad g_i(\ldots X(f(z))\ldots) \\ & \vdots \end{cases} \qquad F(i,j,k) = \begin{cases} \{i,j,k \mid k=0\} & : a_{i,j} \\ \{i,j,k \mid i=j=k\} & : F(i,j,k-1)* \\ \{i,j,k \mid i=k\neq j\} & : F(k,k,k)\otimes F(i,j,k) \\ \{i,j,k \mid j=k\neq i\} & : F(i,j,k-1)\otimes F(k,k,k) \\ \{i,j,k \mid i\neq k; j\neq k\} & : F(i,j,k-1)\oplus \\ & \quad (F(i,k,k)\otimes F(k,j,k-1)) \end{cases}$$

≻ Algebraic and syntax transformations (polyhedral model framework, compiler, scheduler, symbolic analysis, ...)
≻ Graph based approaches easily apply to SRE through the underlying dependence graph ⇒ systematic synthesis !

Scheduling a system of recurrence equations onto p processors is the task of finding a valid *timing function* T (i.e. $T(z) > T(f(z))$) and an allocation function A such that $[T(z_1) = T(z_2)] \Rightarrow [A(z_1) \neq A(z_2)]$.



Theorem 1. *Let $G_1 = (X_1, \Gamma_1)$ and $G_1 = (X_2, \Gamma_2)$ two isomorphic graphs, and let φ an isomorphism from G_2 to G_1. If (t, a) is a valid schedule of G_1, where t is a timing function and a an allocation function, then $(t \circ \varphi, a \circ \varphi)$ is a valid schedule of G_2.*



Theorem 2. *Let $G = (X, A)$ be a directed acyclic graph. If G is self isomorphic with the decomposition $X = X_1 \cup X_2 \cup \cdots \cup X_\gamma$ and the set of isomorphisms $\{\varphi_k, k = 2, \cdots, \gamma\}$, then the schedule specified by the timing function t and the allocation function a given by (6-7) is valid.*

Applications: sorting, APP, Cholesky, tensor product of matrices

Research topics

- Fundamental Aspects of Algorithms and Complexity (Formalism, proof, quantification, classification, ...)
- Discrete Dynamical Systems (Chip Firing Game, Periodicity, Garden of Eden, Invariants, ...)
- Parallel Scheduling (SPMD, SIMD, Systolic, ...)
- Polyhedral model (recurrences equations, re-indexation, systematic synthesis, code derivation, ...)
- Efficient Parallel Programming (Standard Parallel Machines et (Heterogeneous/Clustered) multicores machines)
- Program Transformations (Source-to-source framework aiming at monitoring parallel implementations)
- Power Aware Computing (Energy Efficient Embedded Programs)
- Applied Mathematics and Operation Research (Mathematical modeling, Optimization, Linear Algebra, Matrix Computation)

Research topics

- Fundamental Aspects of Algorithms and Complexity (Formalism, proof, quantification, classification, ...)
- Discrete Dynamical Systems (Chip Firing Game, Periodicity, Garden of Eden, Invariants, ...)
- Parallel Scheduling (SPMD, SIMD, Systolic, ...)
- Polyhedral model (recurrences equations, re-indexation, systematic synthesis, code derivation, ...)
- Efficient Parallel Programming (Standard Parallel Machines et (Heterogeneous/Clustered) multicores machines)
- Program Transformations (Source-to-source framework aiming at monitoring parallel implementations)
- Power Aware Computing (Energy Efficient Embedded Programs)
- Applied Mathematics and Operation Research (Mathematical modeling, Optimization, Linear Algebra, Matrix Computation)

Research topics

- Fundamental Aspects of Algorithms and Complexity (Formalism, proof, quantification, classification, ...)
- Discrete Dynamical Systems (Chip Firing Game, Periodicity, Garden of Eden, Invariants, ...)
- Parallel Scheduling (SPMD, SIMD, Systolic, ...)
- Polyhedral model (recurrences equations, re-indexation, systematic synthesis, code derivation, ...)
- Efficient Parallel Programming (Standard Parallel Machines et (Heterogeneous/Clustered) multicores machines)
- Program Transformations (Source-to-source framework aiming at monitoring parallel implementations)
- Power Aware Computing (Energy Efficient Embedded Programs)
- Applied Mathematics and Operation Research (Mathematical modeling, Optimization, Linear Algebra, Matrix Computation)

Research topics

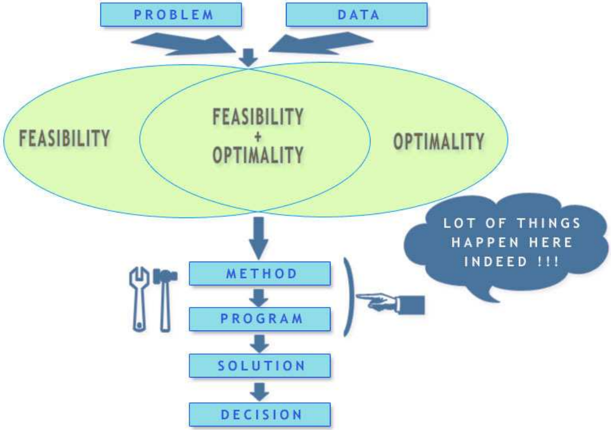
- Fundamental Aspects of Algorithms and Complexity (Formalism, proof, quantification, classification, ...)
- Discrete Dynamical Systems (Chip Firing Game, Periodicity, Garden of Eden, Invariants, ...)
- Parallel Scheduling (SPMD, SIMD, Systolic, ...)
- Polyhedral model (recurrences equations, re-indexation, systematic synthesis, code derivation, ...)
- Efficient Parallel Programming (Standard Parallel Machines et (Heterogeneous/Clustered) multicores machines)
- Program Transformations (Source-to-source framework aiming at monitoring parallel implementations)
- Power Aware Computing (Energy Efficient Embedded Programs)
- Applied Mathematics and Operation Research (Mathematical modeling, Optimization, Linear Algebra, Matrix Computation)

Topics or Aspects to Explore

A deep investigation of the following points are potentially rewarding:

- implement our recursive scheduling method as a module of a polyhedral model based framework.
- develop robust and user-friendly source-to-source tools to monitor parallel programming on accelerators based systems.
- provide specialized mathematical or communication libraries for specific architectures.
- clarify which algorithmic paradigms or programming model yield the best benefit of emergent architectures.
- participate in multidisciplinary efforts for solving large scale problems efficiently on parallel systems.

Operation Research Overview



Operation Research Overview

Remark: Computationally challenging aspect of problems are referred as large-scale problems. This could come either from the tremendous search space or input data, or from the structural intractability of the problem.

Some Illustrative Instances

- P-MEDIAN

  Open P facilities relative to N customers with minimum sum of the shortest demand weighted distance between customers and facilities.

  Applications: cluster analysis, facility location, services (telecommunication, emergency, ...)
  $N = 20$ and $P = 5$ yields $15,505$ possibilities
  $N = 50$ and $P = 10$ yields $\approx 10^{10}$ possibilities

$$
\begin{aligned}
\min_{x,y} \quad & \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{m} x_{ij} = 1, \quad \forall j, \\
& \sum_{i=1}^{m} y_i = p, \\
& x_{ij} \leq y_i, \quad \forall i, j, \\
& x_{ij}, y_i \in \{0, 1\},
\end{aligned}
$$

Some Illustrative Instances

- CREW PAIRING PROBLEM

  Finding a set of round trips (pairings) (starting and ending at the home base) covering all flight legs that the company has to carry out with a **minimal crew cost**.

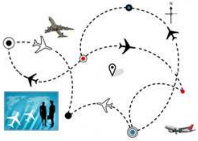
Applications:

CREW SCHEDULING PROBLEM = CREW PAIRING PROBLEM + CREW ASSIGNMENT PROBLEM.

Optimal crew management and best recovery in case of disruption.

- As difficult as the SET COVERING PROBLEM, which is NP-COMPLETE (Karp)
- The operational crew cost can show up more than 5% over predictions in a month (≈ million $)

$$\begin{aligned} &\min_x cx \\ &Ax = 1 \\ &x \in \{0,1\}^n \end{aligned}$$



Some Illustrative Instances

- TRAVELING SALESMAN PROBLEM

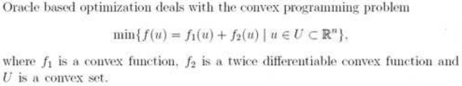
  Given a list of cities and their pairwise distances, the task is to find a **shortest tour** that visits each city exactly once.

  ## Applications:

  Transportation, logistic, genome sequencing, benchmark for optimization methods, ...

  - As difficult as the HAMILTONIAN CYCLE PROBLEM, which is NP-COMPLETE (Karp)
  - Please, forget about brute force approach! (**16 cities ⇒ 653 837 184 000 possibilities**)
  - Modern optimization method have shown optimistic performances on practical instances
  - Competitive solutions are **parallel implementation of powerful optimization methods**
  - TSP (VLSI-Bell Labs) of size **85 900** solved in **1.5 year** (2004-2006) using a cluster of 96 2.8 GHz Intel Xeon and 32 2.4 GHz AMD Opteron connected with 100 MB ethernet.

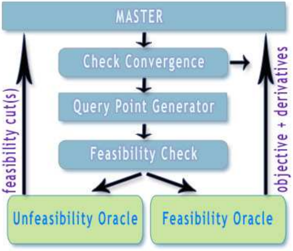
Continuous Optimization Scheme



Oracle based optimization deals with the convex programming problem

$$\min\{f(u) = f_1(u) + f_2(u) \mid u \in U \subset \mathbb{R}^n\},$$

where f_1 is a convex function, f_2 is a twice differentiable convex function and U is a convex set.

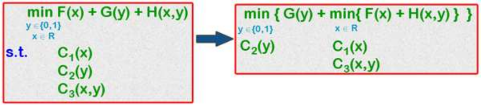


Interesting points are:

- improve the accuracy of the lower bound estimation
- invest on convergence acceleration strategies
- parallelization of the linear algebra kernel of existing methods
- parallelization of the function and/or the derivative evaluations
- parallel implementation of decomposition methods (projection, multi-objectives)

Discrete Optimization↔Continuous Optimization

- Bender decomposition



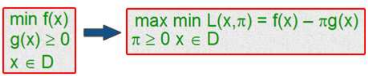
$$\min_{\substack{y\in\{0,1\}\\ x\in R}} F(x)+G(y)+H(x,y)$$

s.t. $C_1(x)$ $C_2(y)$ $C_3(x,y)$

$$\min_{y\in\{0,1\}}\{\, G(y)+\min_{x\in R}\{\, F(x)+H(x,y)\,\}\,\}$$

$C_2(y)$ $C_1(x)$ $C_3(x,y)$

- Lagrangian relaxation



$\min f(x)$
$g(x)\geq 0$
$x\in D$

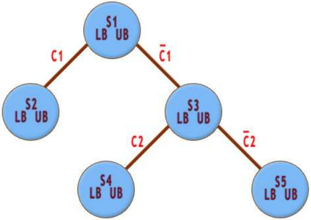
$$\max_{\pi\geq 0}\min_{x\in D} L(x,\pi)=f(x)-\pi g(x)$$

The idea behind is to “remove” the offending constraint by incorporating its violation in the objective.

Examples

- Constrained shortest path
- Time optimal scheduling under memory/energy constraint
- Portfolio selection under cardinality constraints

Branch and Bound



- efficient parallel heuristics (upper bounds)
- efficient and generic multi-thread implementation of the global branch and bound scheme
- parallel branching with N child nodes ($N \geq 2$ and non constant)
- workload balance, synchronization, scalability, and high efficiency

Aspects to Explore

A deep investigation of the following points are potentially rewarding:

- modifying existing methods for better exposure of the parallelism.
- parallel approximation algorithms.
- parallel multidirectional search strategies.
- provide more efficient parallelization of interior point method.
- new paradigms

END & QUESTIONS