Towards Compositional and Generative Tensor Optimizations

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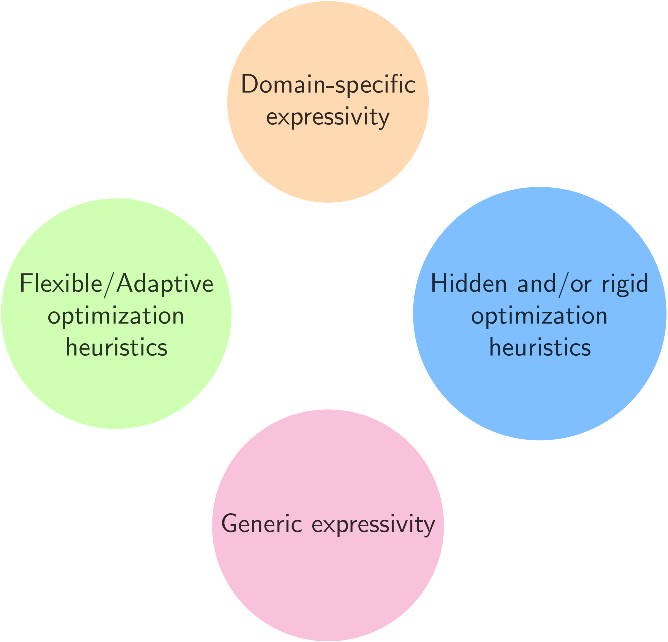
Tensor Computations

- Underlying data structure: N-dimensional array

Applications in numerical applications

- Quantum chemistry
- Machine learning
- Big data
- Computational fluid dynamics

Frameworks for Optimizations for Tensor Computations



Tensors in Computational Fluid Dynamics

Characteristics

- 3 to 4 dimensions nesting
- Few iterations per dimension (e.g., 13 iterations)
- Tensor contractions, outer products, entrywise multiplications
- Same computation for each element of a mesh

Inverse Helmholtz [7]

$$t_{ijk} = \sum_{l,m,n} A^T_{kn} \cdot A^T_{jm} \cdot A^T_{il} \cdot u_{lmn}$$

$$p_{ijk} = D_{ijk} \cdot t_{ijk}$$

$$v_{ijk} = \sum_{l,m,n} A_{kn} \cdot A_{jm} \cdot A_{il} \cdot p_{lmn}$$

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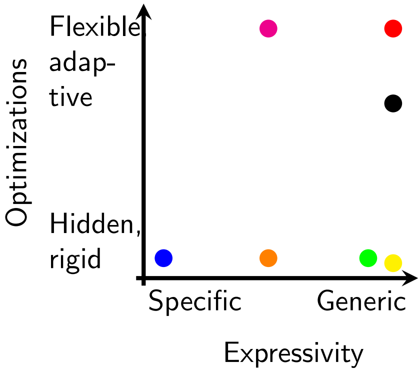
Search space for optimizations may include

- Evaluation order of tensor contractions
- Fusions
- Interchanges
- Transpositions
- Vectorization
- Collapsing
- Unrolling

Implementing CFD Kernels in Existing Frameworks



Chill ● [6]

Pluto ● [5]

TensorFlow ● [3]

TVM ● [2]

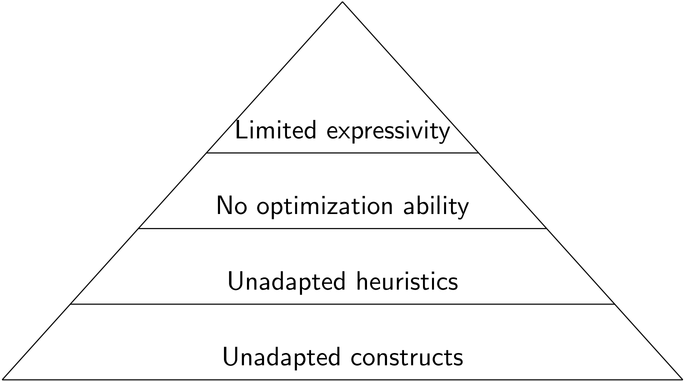
Tensor Contraction Engine ● [4]

Numpy ● [1]

Tensor Algebra Compiler ● [8]

Implementing CFD Kernels in Existing Frameworks

We encounter different levels of limitations

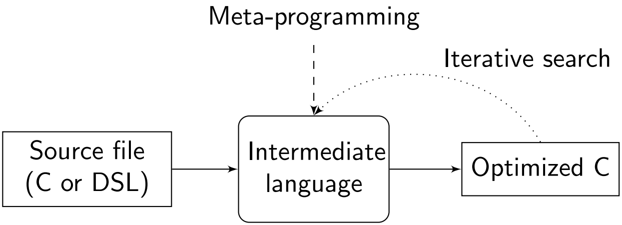


Our contribution

An intermediate language with building blocks for declaring:

- Tensor computations
- Optimization heuristics

Arrays, tensor operators, iterators and loop transformations as first class citizens.



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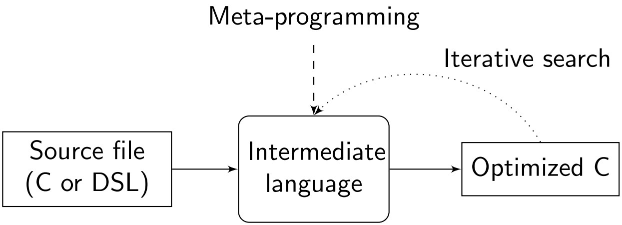


CFD kernels share common tensor operations with other domains

- **We want enough flexibility and genericity (at least for tensor-based applications) to be reused in other domains.**

Inverse Helmholtz by Example

$$t_{ijk} = \sum_{l,m,n} A^T_{kn} \cdot A^T_{jm} \cdot A^T_{il} \cdot u_{lmn}$$

$$p_{ijk} = D_{ijk} \cdot t_{ijk}$$

$$v_{ijk} = \sum_{l,m,n} A_{kn} \cdot A_{jm} \cdot A_{il} \cdot p_{lmn}$$

Step 1: Declaring tensor computations

```
A = array(2, double, [N, N])
u = array(3, double, [N, N, N])
D = array(3, double, [N, N, N])
At = vtranspose(A, 1, 2)
tmp1 = contract(At, u, [2, 1])
tmp2 = contract(At, tmp1, [2, 2])
tmp3 = contract(At, tmp2, [2, 3])
tmp4 = entrywise(D, tmp3)
tmp5 = contract(A, tmp4, [2, 1])
tmp6 = contract(A, tmp5, [2, 2])
v = contract(A, tmp6, [2, 3])
```

Inverse Helmholtz by Example

Step 2: Associating iterators to computations

```
i1 = iterator(0, N, 1)
i2 = iterator(0, N, 1)
# ... other iterator declarations

build(D, [td1, td2, td3])
build(tmp1, [i1, i2, i3, i4])
## Also applies to tmp2, ..., tmp6
build(v, [k12, k22, k32, k42])
```

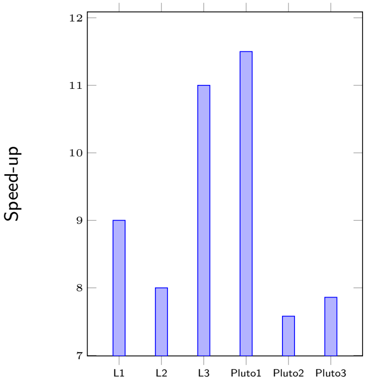
Inverse Helmholtz by Example

Step 3: Applying transformations

```
interchange(i4, i3)
interchange(i4, i2)
interchange(j2, j1)
interchange(j1, j4)
```

Inverse Helmholtz by Example

Example of results from different heuristics



- Mesh size: 750; data size: 33.
- Baseline: sequential execution.
- Machine: 24-core Intel(R) Xeon(R) CPU E5-2680 v3 @ 2.50GHz (Haswell)

- Variant L1: Loop interchanges only + parallelization;
- Variant L2: Loop interchanges + data transpositions of tensor A + parallelization;
- Variant L3: Loop interchanges + data transpositions of tensors tmp1, ..., tmp6 + parallelization.
- Pluto1: Loop interchanges + parallelization + vectorization;
- Pluto2: Loop interchanges + partial fusions + vectorization;
- Pluto3: Loop interchanges + maximum fusions + vectorization;

Conclusion

- Cross-domain building-blocks

  $\rightarrow$ One intermediate language to rule them all flexibly
- Possibility to assess different variants

  $\rightarrow$ Through meta-programming or auto-tuning techniques

Ongoing work

- Syntax refinement
- Formal semantics
- Applications to other domains

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