The MMT Language and System
The LATIN Logic Atlas

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Deducteam-Kwarc workshop, April 12 2013

MMT Vision

- Universal framework for mathematical-logical content
- Close relatives
  - LF, Isabelle: but more universal, knowledge management, more system integration
  - OMDoc/OpenMath: but formal semantics, automation
- Typical use case
  1. define a logical framework in MMT — e.g., LF
  2. use it to define a logic in MMT — e.g., HOL
  3. optionally: write and register plugins — e.g., type checking
  4. MMT induces a system for that logic

     provides logical and knowledge management services

     handles system integration

Statistics

- MMT language
  - 5 years of development (with Michael)
  - $\sim$ 100 pages write-up
- MMT API
  - 5 years of development (with various students)
  - 30,000 lines of Scala code
  - $\sim$ 10 papers on individual aspects

Example: small scale

- **Little theories**: state every definition/theorem/algorithm in the smallest possible theory/logic/logical framework
- **Theory morphisms**: transport results across theories/logics/logical frameworks

```
theory Types { type }
theory LF {include Types, Π, →, λ, @ }

theory Logic meta LF {o: type, ded : o → type }
theory FOL meta LF {
  include Logic
  u: type. ⇒: o → o → o, ...
}

theory Magma meta FOL { ∘: u → u → u }
⋮
theory Ring meta FOL {
  additive: CommutativeGroup
  multiplicative: Semigroup
  ...
}
```

Example: large scale

- LATIN atlas of logics: highly interconnected network of logic formalizations
- Written in MMT/LF using Twelf
- 4 years, $\sim$ 10 authors, $\sim$ 1000 modules
- Focus on breadth (= many formal systems represented), not so much depth (= theorems in particular systems)
- Each logic root for library of that logic
- Each edge yields library translation functor

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Important meta-result: the logical framework should be flexible

MMT Design Methodology

1. Choose a typical problem

   logical: e.g., type reconstruction, reflection

   MKM: e.g., change management, querying

2. Survey and analyze the existing solutions
3. Differentiate between foundation-specific and foundation-independent definitions/theorems/algorithms
4. Integrate the foundation-independent aspects into MMT

   language and system

5. Define interfaces to supply the logic-specific aspects

   formal and plugin interfaces

6. Repeat

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3. We can fix and implement a logical framework
   then define many logics in it the foundation, e.g., LF
4. We can fix and implement meta-framework
   then define many logical frameworks in it
   foundation-independence, e.g., MMT

The Promise and Danger of Abstraction

- Abstraction chain

  $$\textit{theory} \to \textit{logic} \to \textit{foundation} \to \textit{MMT}$$

- Promises: high-level results — generic, reusable!
  - intuitions, documentation, teaching
  - definitions, meta-theorems
  - algorithms, implementations
  - knowledge management
- Dangers: loss of precision — general abstract nonsense?
  - how useful are the abstract results?

    are the deep results foundation-specific?
  - how much work (if any) is needed for specialization?

    hide the framework from the user

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  - editing, parsing
  - change management
  - project management, distribution
  - search, querying
  - interactive browsing

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  - change management
  - project management, distribution
  - search, querying
  - interactive browsing
- Logical services?
  - module system — Yes!
  - type reconstruction — Yes?
  - computation — current work
  - theorem proving — future work

Features of MMT

few primitives . . . that unify different domain concepts

JaT judgments as types, proofs as terms

unifies expressions and derivations

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MS module system (little theories)
unifies inheritance and representation theorems

MaM models as morphisms (categorical logic)
unifies syntactical translations and semantic interpretations

Features of MMT (2)

- current work: declaration patterns
  unifies declarations and extension principles
- current work: induction, coinduction
  unifies multiple constructions/reasoning principles
- current work: reflection
  unifies meta- and object level

  for example, module system
  - meta-level: MMT theories
  - object-level: record types

The MMT System

Application-independence

1. data structures
2. logical and knowledge management services
3. individual applications

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2. logical and knowledge management services
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Advantages

- flexibility
- no compromises, hacks
- high code reuse

Disadvantages

- no running system bad for talks like this one
- starting MMT gives only an empty environment

  no single name defined

Implementing Services in MMT

- Isolate functionality into services
- Integrate interfaces with core
- Then do 2 implementation approaches
  - plugin interfaces — arbitrary implementations
  - generic implementation — parametrized as declaratively as possible

Logical Services Example: Type reconstruction

- type reconstruction
  - input: judgment with unknown variables

$$\lambda_{n:?}\lambda_{l:?}\texttt{cons}\ ?\ c\ l \qquad \Leftarrow \qquad \Pi_{n:?}\mathit{list}\ n \to ?$$

  - output: derivation of judgment and solution for variables

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1 month full time work

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- generic implementation
  - parametrized by sets of rules — ~ 8 rule types
  - origin of rules up to plugins

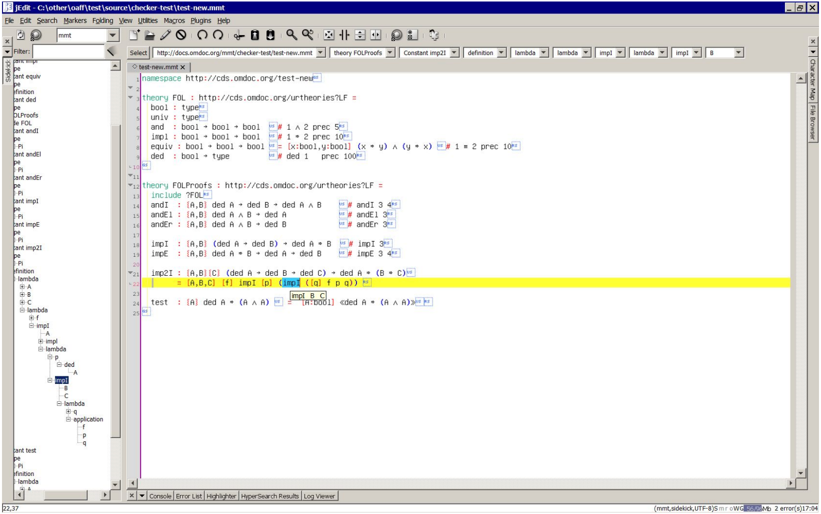
LF plugins provides ~ 10 rules, each a few lines of code

Application Example: Editing

- IDE like based on MMT projects
- jEdit text editor with MMT as plugin
- Generic default implementations for parsing
  - outer syntax: extensible through keyword handlers
  - inner syntax: extensible through notation language
- Cross-references MMT data structures $\leftrightarrow$ source locations
  - outline view
  - hyperlinks (= click on operator, jump to declaration/definition)
  - context-sensitive auto-completion: show identifiers that
    - are in scope
    - have the right type

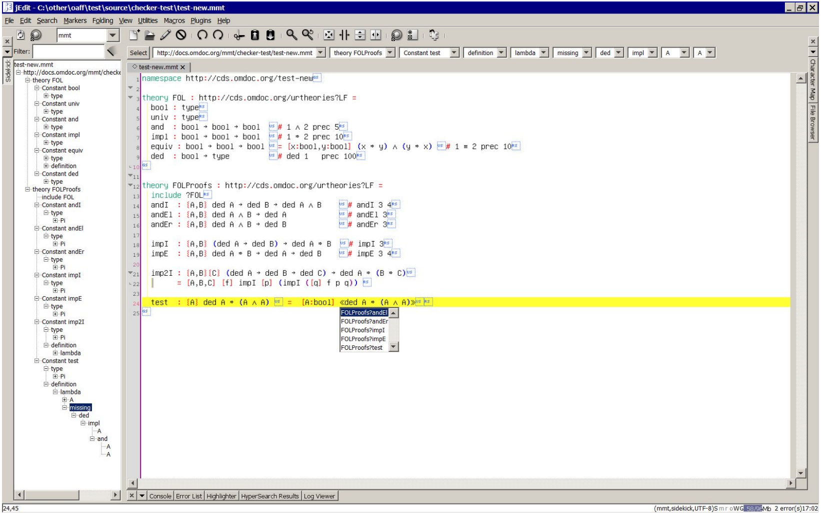
Application Example: Editing

Example feature: pop up shows reconstructed arguments



Application Example: Editing

Example feature: auto-completion shows only identifiers that are in scope and have the right type



Application Example: LaTeX Integration

- Unified document format LaTeX + MMT
- Processed by LaTeX
- MMT-relevant aspects represented in special macros
  sent to MMT via HTTP during compilation
- LaTeX queries MMT at run time via HTTP
  1. parse
  2. type reconstruct
  3. generate high-quality LaTeX cross-references, tooltips

Application Example: LaTeX Integration

Example feature

- upper part: LaTeXsource for the item on associativity
- lower part: pdf after compiling with LaTeX-MMT
- type argument M of equality symbol is inferred and added by MMT

```
\begin{mmtscope}
 For all \mmtvar{x}{in M},\mmtvar{y}{in M},\mmtvar{z}{in M}
 it holds that !(x * y) * z = x * (y * z)!
\end{mmtscope}
```

A *monoid* is a tuple $(M, \circ, e)$ where

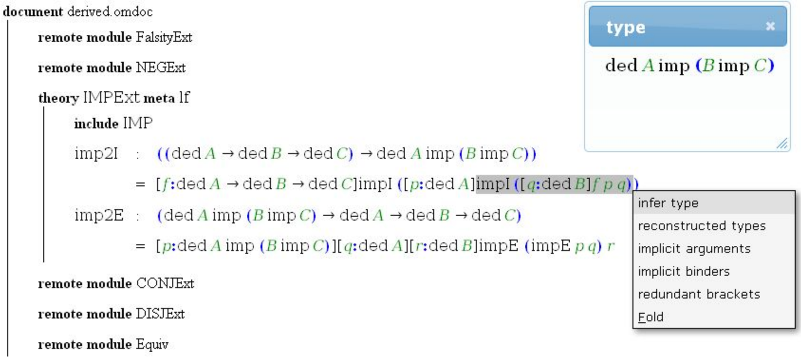
- M is a sort, called the universe.
- $\circ$ is a binary function on M.
- e is a distinguished element of M, the unit.

such that the following axioms hold:

- For all x,y,z it holds that $(x \circ y) \circ z =_M x \circ (y \circ z)$
- For all x it holds that $x \circ e =_M x$ and $e \circ x =_M x$.

Application example: Interactive Browsing

- MMT API exposed through HTTP server
- Javascript/Ajax for interactive browsing of MMT projects
  e.g., definition lookup, dynamic type inference
- Interactive graph view
- Immediate editing ongoing work



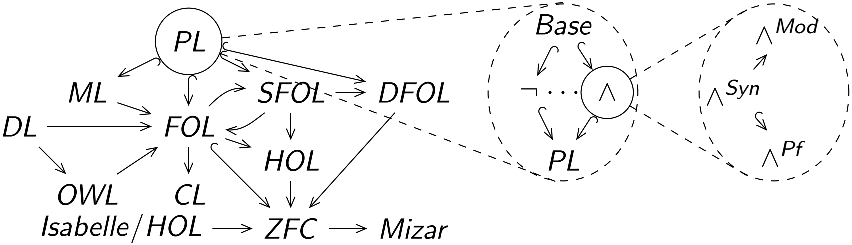
The LATIN Library

- Joint project between DFKI Bremen and Jacobs Univ. Bremen
- Development of an atlas of logics and logic translations
  - reference catalog of standardized logics
  - documentation platform
- All parts of a logic represented in MMT/LF
- Easy part
  - Logical syntax proof theory as MMT/LF theories
  - Judgments as types, higher-order abstract syntax
- Hard part
  - Foundations of mathematics as LF signatures
  - Models as morphisms from the syntax to the foundation

Current State

- 700 little theories including
  - propositional, (unsorted, sorted, dependently-sorted) first-order, higher-order, common, modal, description, linear logic
  - λ-cube, Curry and Church-style type theories
  - ZFC set theory, Mizar's set theory, Isabelle/HOL
  - category theory
- 500 little morphisms including
  - relativization of quantifiers from sorted first-order, modal, and description logics to unsorted first-order logic
  - negative translation from classical to intuitionistic logic
  - translation from type theory to set theory
  - translations between ZFC, Mizar, Isabelle/HOL
  - Curry-Howard correspondence between logic, type theory, and category theory

Little Theories in LATIN



Representing Logics in LATIN

- L^{syn}: Syntax of L: connectives, quantifiers, etc.
  e.g., $\Rightarrow: o \to o \to o$
- L^{pf}: Proof theory of L: judgments, proof rules
  e.g., $impE \ : \ \mathtt{ded}\,(A \Rightarrow B) \to \mathtt{ded}\,A \to \mathtt{ded}\,B$
- L^{mod}: Model theory of L in terms of foundation $\mathcal{F}$
  e.g., $univ : set$, $nonempty : true\,(univ \neq \emptyset)$

$$\begin{tikzcd}
 & L^{pf} \\
L^{syn} \arrow[ur] \arrow[dr] & \\
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Example

- FOL^{Syn}: $i : type$, $o : type$, $\mathtt{ded} : o \to type$, $\neg$, $\wedge$, ...
- FOL^{Pf}: $\neg I$, $\neg E$, $\wedge E_l$, $\wedge E_r$, $\wedge I$, ...
- ZFC: $set : type$, $prop : type$, $true : prop \to type$, $\varnothing : set$, ...
- FOL^{Mod}: $univ : set$, $nonempty : true\ (univ \neq \varnothing)$
- FOL^{mod}: $i := univ$, $o := \{\varnothing, \{\varnothing\}\}$, $\mathtt{ded} := \lambda_p(p \doteq \{\varnothing\})$

$$
\begin{tikzcd}
& FOL^{Pf} \\
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Representing Logics and Models

$$\begin{tikzcd}
\mathcal{F} \arrow[r, "id_{\mathcal{F}}"] \arrow[d, hook] & \mathcal{F} \\
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L \arrow[u, "L^{mod}"'] \arrow[r, hook] & \Sigma \arrow[u, "\Sigma^{mod}"']
\end{tikzcd}$$

L encodes syntax and proof theory
$\mathcal{F}$ encodes foundation of mathematics
L^{Mod} axiomatizes models
L^{mod} interprets syntax in model

Σ encodes a theory of L,
extends L with functions, axioms, etc.
Σ^{Mod} correspondingly extends L^{Mod}
Σ^{mod} interprets syntax in model

M encodes a model of Σ,
interprets free symbols of L^{Mod} and Σ^{Mod}
in terms of $\mathcal{F}$

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Conclusion

- Very general, customizable framework — goal: universal
- Foundation-independent representation language — integrates best primitives
- Interface for logical and knowledge management services
- Rapid prototyping logic systems — scalable
- Interesting for
  - less well-supported logics
  - new, changing logics
  - generic applications/services
  - system integration/combination