

# The Energy/Frequency Convexity Rule of Energy Consumption for Programs: Modeling, Thermosensitivity, and Applications

Karel De Vogeleer

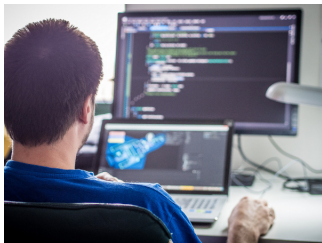
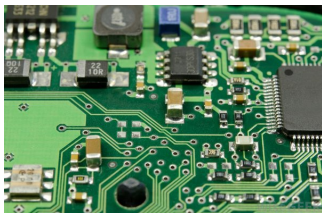
Ph.D. defense  
September 4th, 2015







# A Green IT Thinking



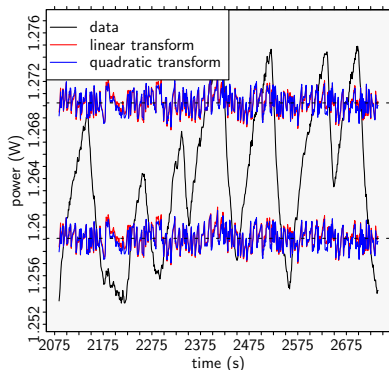
- Off-line, including
  - ▶ transistor design,
  - ▶ circuit design,
  - ▶ architecture,
  - ▶ software design,
  - ▶ software coding,
  - ▶ compiler optimization;
- on-line, including
  - ▶ system reconfiguration,
  - ▶ compiler optimization,
  - ▶ context placement.

# Contributions

- Energy consumption analysis for computer systems:
  - ▶ analytical model,
  - ▶ Energy/Frequency Convexity Rule,
  - ▶ supportive measurement data;
- temperature/power relationship demystified:
  - ▶ supportive measurement data,
  - ▶ guidelines for power measurement;
- transient thermal model for microprocessors:
  - ▶ analytical model including radiation,
  - ▶ approximations,
  - ▶ applicability analysis.

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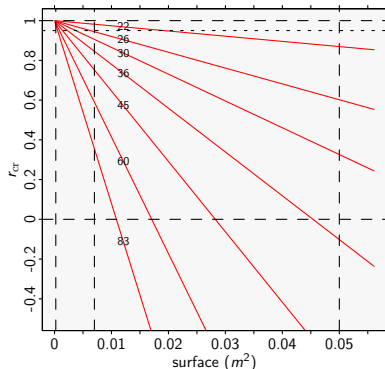


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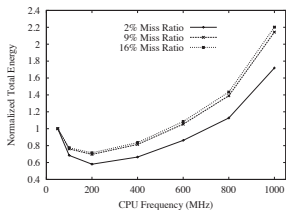
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# Presentation's Outline

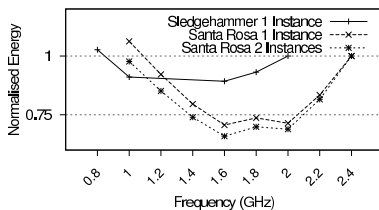
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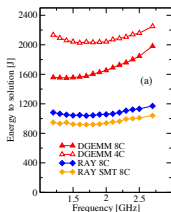
# Preliminary Evidence of Energy/Frequency Curves



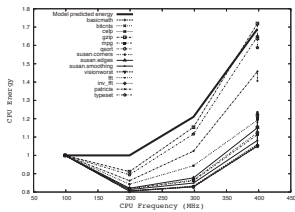
(a) Fan et al. [1]



(b) Le Sueur and Heiser [3]



(c) Hager et al. [2]



(d) Snowdon et al. [4]

# System Energy Consumption Model ( $E_{\text{sys}}$ )

- System's energy consumption  $E_{\text{sys}}$  definition

$$E_{\text{sys}} = \int_0^{\Delta t} P_{\text{sys}} dt$$

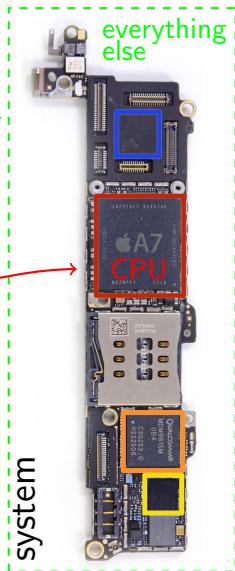
$$= \int_0^{\Delta t} (P_{\text{cpu}} + P_{\text{back}}) dt;$$

- Examples of  $P_{\text{back}}$  include:

- ▶ LCD screen,
- ▶ radio interface,
- ▶ sensors (e.g. GPS);

- If  $P_{\text{cpu}}$  and  $P_{\text{back}}$  are constant over  $\Delta t$ :

$$E_{\text{sys}} = (P_{\text{cpu}} + P_{\text{back}}) \cdot \Delta t.$$



## Microprocessor Power Model

CPU power  $P_{\text{cpu}}$  consists of:

- dynamic power  $P_{\text{dyn}}$ ,
- leakage current  $P_{\text{leak}}$ ,
- short-circuit current  $P_{\text{sc}}$ ,

$$\begin{aligned}
 P_{\text{cpu}} &= P_{\text{dyn}} + P_{\text{leak}} + P_{\text{sc}} \\
 &= (1 + \gamma V) \cdot \eta \alpha C V^2 f \\
 &= (1 + \gamma V) \cdot \xi V^2 f.
 \end{aligned}$$

## Execution Time Model

Execution time  $\Delta t$  depends on:

- $cc_b$  code size in clock cycles,
- $f$  CPU clock frequency,
- $f_k$  frequency thieves,
- $\beta$  slack time per clock cycle,

$$\Delta t = cc_b \left( \frac{1}{f - f_k} + \beta \right).$$

# Optimal Clock Frequency ( $f_{\text{opt}}$ )

- System's energy consumption model

$$\begin{aligned} E_{\text{sys}}(f) &= (P_{\text{cpu}} + P_{\text{back}}) \cdot \Delta t \\ &= ([1 + \gamma V] \xi V^2 f + P_{\text{back}}) \cdot c c_b \left( \frac{1}{f - f_k} + \beta \right), \end{aligned}$$

where  $\{\gamma, \xi, P_{\text{back}}, c c_b, f_k, \beta\} \in \mathbb{R}^+$ ;

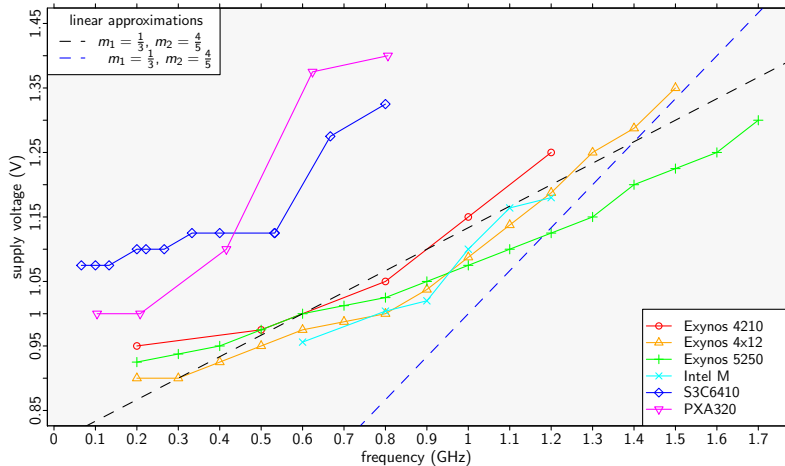
- a single minimum for  $E_{\text{sys}}(f)$  exists at  $f_{\text{opt}}$  when

$$\left( \frac{\partial E_{\text{sys}}}{\partial f} \right)_{f=f_{\text{opt}}} = 0, \quad \text{and} \quad \frac{\partial^2 E_{\text{sys}}}{\partial f^2} > 0 \quad \text{holds};$$

- $V$  is approximately an affine map of  $f$ :  $V \rightarrow m_2 f + m_1$ .

# Supply Voltage/Frequency Relationship

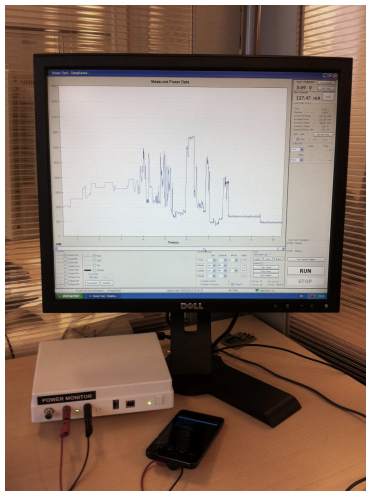
A linear trend between  $V$  and  $f$  is observed:  $V = m_2 f + m_1$ .





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# Benchmark and Testbed

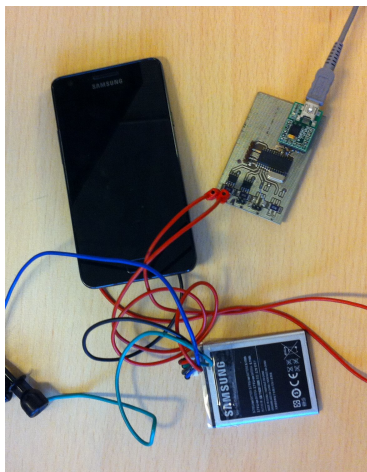


- **Benchmark:** bit-reverse algorithm, part of the DFT algorithm:

```
void bitreverse_gold_rader
    (int N, complex *data) {
    int n = N, nm1 = n-1;
    int i = 0, j = 0;
    for (; i < nm1; i++) {
        int k = n >> 1;
        if (i < j) {
            complex temp = data[i];
            data[i] = data[j];
            data[j] = temp;
        }
        while (k <= j) {
            j -= k; k >>= 1;
        }
        j += k ;
    }
}
```

- **testbed:** Samsung Galaxy SII;
- **power Measurement:** Monsoon.

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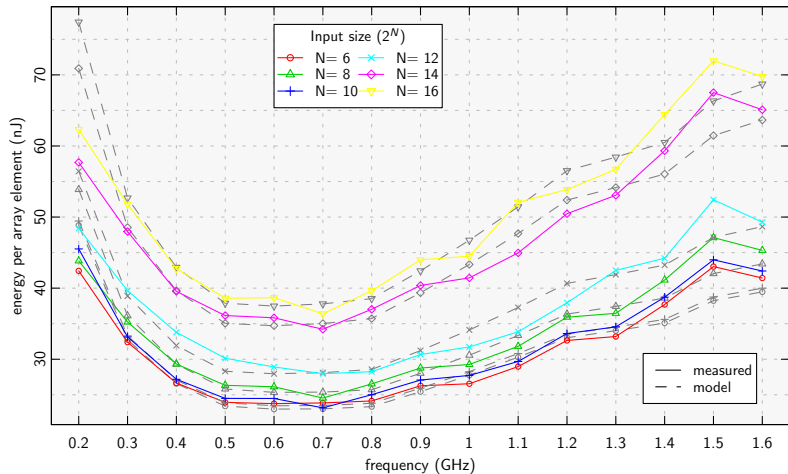
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# The Energy/Frequency Convexity Rule

Energy consumption versus CPU clock frequency shows convex properties.



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# Energy Model's Parameter Sensitivity Analysis

- Energy consumption model under analysis:

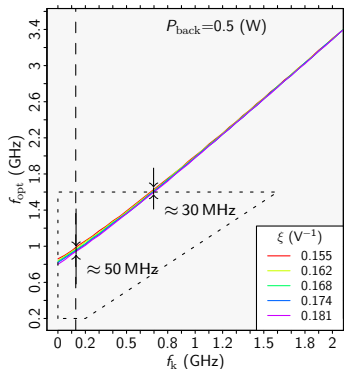
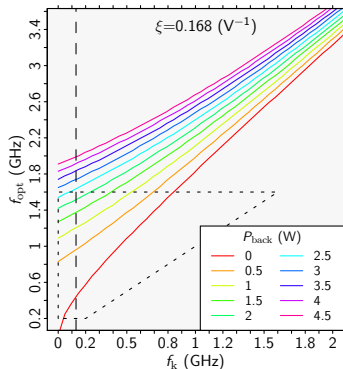
$$E_{\text{sys}} = ([1 + \gamma V] \cdot \xi V^2 f + P_{\text{back}}) \cdot c_{\text{cb}} \left( \frac{1}{f - f_{\text{k}}} + \beta \right),$$

$$\left( \frac{\partial E_{\text{sys}}}{\partial f} \right)_{f=f_{\text{opt}}} = 0;$$

- The aim is to find the conditions under which  $f_{\text{opt}}$  is exploitable;
- The following parameters will be looked at in more detail:
  - ▶ frequency thieves (overhead)  $f_{\text{k}}$ ,
  - ▶ background power  $P_{\text{back}}$ ,
  - ▶ power gain  $\xi$ ,
  - ▶ temperature  $\gamma(T)$ ;
- Analysis based on energy profile of the Exynos 4210.

# Influence of *frequency thieves* $f_k$ on $f_{\text{opt}}$

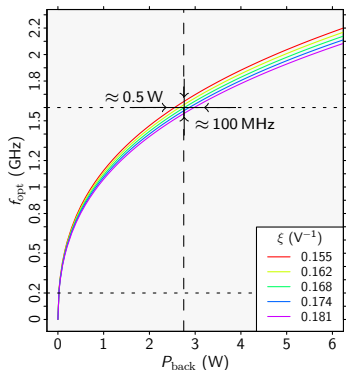
$$E_{\text{sys}} = ([1 + \gamma V] \cdot \xi V^2 f + P_{\text{back}}) \cdot c c_b \left( \frac{1}{f - f_k} + \beta \right)$$

(e)  $f_{\text{opt}}(f_k, \xi)$ (f)  $f_{\text{opt}}(f_k, P_{\text{back}})$

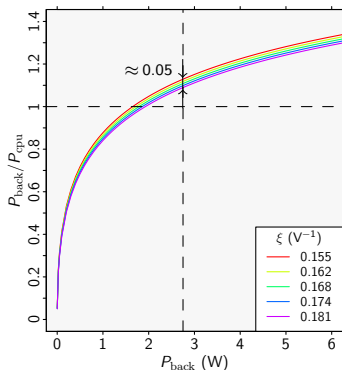


# Influence of *Background Power* $P_{\text{back}}$ on $f_{\text{opt}}$

$$E_{\text{sys}} = ([1 + \gamma V] \cdot \xi V^2 f + P_{\text{back}}) \cdot c c_b \left( \frac{1}{f - f_k} + \beta \right)$$



(g)  $f_{\text{opt}}(P_{\text{back}}, \xi)$

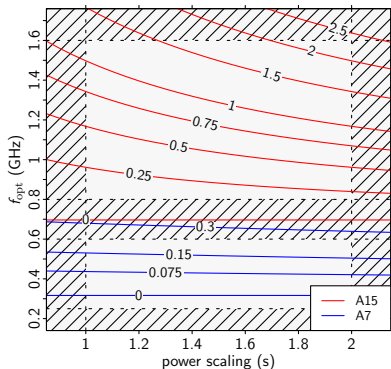


(h)  $P_{\text{back}}/P_{\text{cpu}}$  ratio at  $f_{\text{opt}}$

# Influence of *Power Gain* $\xi(s)$ on $f_{\text{opt}}$

$$E_{\text{sys}} = ([1 + \gamma V] \cdot \xi V^2 f + P_{\text{back}}) \cdot c c_b \left( \frac{1}{f - f_k} + \beta \right)$$

- Cooperative microprocessors on the same die:
  - power-efficient: Cortex A7,
  - high-performance: Cortex A15;
- $\xi$  is scaled by  $s$  between its lower and upper bound:  $s \in \{1, 2\}$ ;
- Exynos 5410 power model.

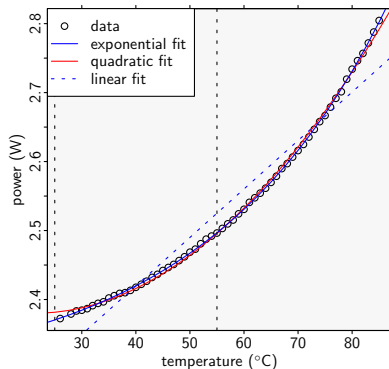


Numbers on the lines represent the background power for that line.

# Influence of *Temperature* $\gamma(T)$ on $f_{\text{opt}}$

$$E_{\text{sys}} = ([1 + \gamma V] \cdot \xi V^2 f + P_{\text{back}}) \cdot c c_b \left( \frac{1}{f - f_k} + \beta \right)$$

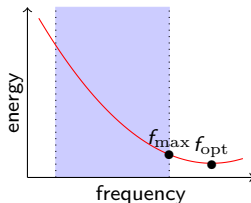
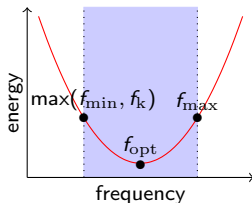
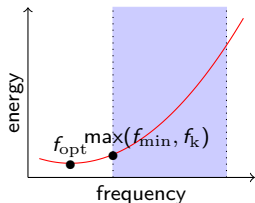
- $\gamma$  is a function of temperature;
- temperature/power model of Exynos 5410 is used;
- temperature/power shows exponential behavior;



$\Delta f_{\text{opt}} \approx 200 \text{ MHz}$  when  $25^\circ\text{C} < T < 85^\circ\text{C}$ .

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# Case Study 1: $f_{\text{opt}}$ Classification

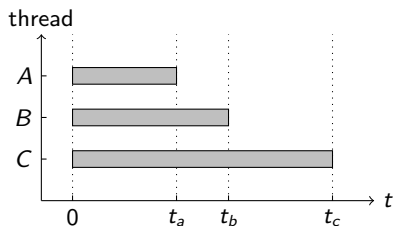


1	$\max(f_{\text{min}}, f_k)$ $f_{\text{max}}$	$\leq$	$f_{\text{opt}}$	$<$	$\max(f_{\text{min}}, f_k)$	the slower, the better chase $f_{\text{opt}}$ race-to-halt
2		$\leq$	$f_{\text{opt}}$	$\leq$	$f_{\text{max}}$	
3		$<$	$f_{\text{opt}}$	$<$	$f_{\text{opt}}$	

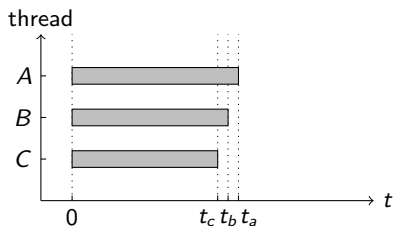
## Case Study 2: $f_{\text{opt}}$ and Multi-core Code Execution

Clock frequency scheduling schemes:

- ① **on-demand**: binary (high/low) as work arrives;
- ② **selfish**: each core is individually energy optimized;
- ③ **thread-cooperation**: all cores are collectively energy optimized.



(a) default clock scaling



(b) cooperative clock scaling

## Case Study 2: $f_{\text{opt}}$ and Multi-core Code Execution contd.

Problem statement:

- $n$  threads executed in parallel with common deadline  $t_{\text{max}}$ ;
- threads individually clock frequency  $f_i$  scalable;
- $E_{\text{tot}}(f_i) : \mathbb{R}^m \rightarrow \mathbb{R}$  to be minimized over  $f_i$ :

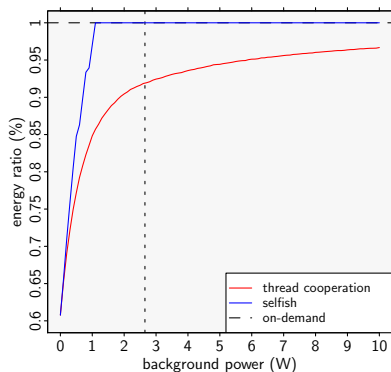
$$E_{\text{tot}}(f_i) = P_{\text{back}} t_{\text{max}} + \sum_{i=0}^n \left[ \frac{CC_{b,i}}{f_i} P^+ + \left( t_{\text{max}} - \frac{CC_{b,i}}{f_i} \right) P^\circ \right],$$

$$\text{subject to } \forall i, \frac{CC_{b,i}}{f_i} \leq t_{\text{max}} \quad \text{and} \quad f_{\text{min}} \leq f_i \leq f_{\text{max}};$$

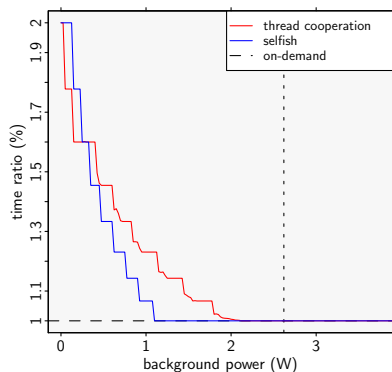
- $\{CC_b, f_i, t_{\text{max}}, P_{\text{back}}, P^\circ, P^+\} \in \mathbb{R}^+$ ;
- active power ( $P^+$ ) and idle power ( $P^\circ$ ) are generated by the Exynos 5410 power model.

# Case Study 2: $f_{opt}$ and Multi-core Code Execution contd.

Performance evaluation of 4 clock frequency scalable parallel threads.



(a) energy

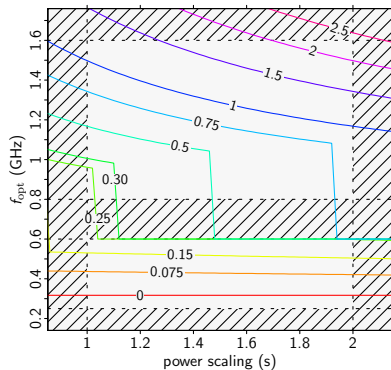
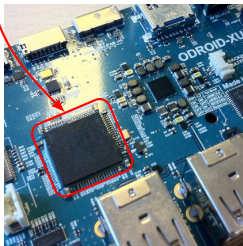


(b) time



# Case Study 3: big-LITTLE Heterogeneous Computing

- Optimal clock frequency for cooperative microprocessors:
  - power-efficient: Cortex A7,
  - high-performance: Cortex A15;
- $f_{opt}$  is chosen on the core yielding best efficiency;
- Exynos 5410 power model used.



Numbers on the lines represent the background power for that line.

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# Conclusion

- System's energy consumption shows convex properties over  $f$ ;
- rules of thumb for an exploitable  $f_{\text{opt}}$ :
  - ▶  $P_{\text{back}}$  should be smaller than  $P_{\text{cpu}}$ ,
  - ▶ overhead ( $f_k$ ) should be limited,
  - ▶ slack time  $\beta$  should be limited,
  - ▶ power profile ( $\xi$ ) has *minimal effect*,
  - ▶ code size ( $cc_b$ ) has *no effect*;
- energy gains could be from 10% up to 50% at fixed temperature;
- temperature/Power relationship shows exponential behavior;
- radiation can be omitted for small devices.



# Future Work

LEARNING



dorrismccomics.com

Including:

- apply results to other domains:
  - ▶ multi-core,
  - ▶ HPC,
  - ▶ clock modulation,
  - ▶ interactive/performance;
- exploit the thermal behavior;
- better understanding of how much energy can practically be gained.

# The Energy/Frequency Convexity Rule of Energy Consumption for Programs: Modeling, Thermosensitivity, and Applications

Karel De Vogeleer

Ph.D. defense  
September 4th, 2015



# Publications

- K. De Vogeleer, G. Memmi, P. Jouvelot, and F. Coelho, "The Energy/Frequency Convexity Rule: modeling and experimental validation on mobile devices," in *Proceedings of the 10th Conference on Parallel Processing and Applied Mathematics*. Springer Verlag, Sep. 2013.
- K. De Vogeleer, G. Memmi, P. Jouvelot, and F. Coelho, "Modeling the temperature bias of power consumption for nanometer-scale CPUs in application processors," in *14th International Conference on Embedded Computer Systems: Architectures, Modeling, and Simulation*, Jul. 2014, pp. 172-180.
- K. De Vogeleer, P. Jouvelot, and G. Memmi, "The impact of surface size on the radiative thermal behavior of embedded systems," CoRR, vol. abs/1410.0628, 2014, (submitted to *IEEE TMC* in 2014).
- K. De Vogeleer, G. Memmi, and P. Jouvelot, "Parameter Sensitivity Analysis of the Energy/Frequency Convexity Rule for Nanometer-scale Application Processors," CoRR, vol. abs/1508.07740, 2015, (in submission to *The Elsevier Journal of Parallel and Distributed Computing*, 2015).

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SNOWDON, D. C., RUOCCO, S., AND HEISER, G. Power management and dynamic voltage scaling: Myths and facts. In *2005 WS Power Aware Real-time Comput.* (New Jersey, USA, Sept. 2005).